Colliding plane waves in general relativity

J. B. GRIFFITHS

Department of Mathematical Sciences
Loughborough University of Technology

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PREFACE

For many years after Einstein proposed his general theory of relativity, only a few exact solutions were known. Today the situation is completely different, and we now have a vast number of such solutions. However, very few are well understood in the sense that they can be clearly interpreted as the fields of real physical sources. The obvious exceptions are the Schwarzschild and Kerr solutions. These have been very thoroughly analysed, and clearly describe the gravitational fields surrounding static and rotating black holes respectively.

In practice, one of the great difficulties of relating the particular features of general relativity to real physical problems, arises from the high degree of non-linearity of the field equations. Although the linearized theory has been used in some applications, its use is severely limited. Many of the most interesting properties of space-time, such as the occurrence of singularities, are consequences of the non-linearity of the equations.

In this book we will be considering one of the most obvious situations in which the effects of the non-linearity of Einstein's equations will be manifest. We will be considering the interaction between two waves. By restricting our attention to somewhat idealized situations, it will be possible to describe some types of wave interaction in terms of exact solutions. Moreover, these solutions have a clear physical interpretation in terms of combinations of gravitational or electromagnetic waves and their interaction.

Much attention has been focused on these problems in recent years. An initial approach to the subject was pioneered by Szekeres (1970, 1972) and Khan and Penrose (1971). More recently, an alternative approach using an analogy with stationary axisymmetric solutions has been exploited by Chandrasekhar and Ferrari (1984) and their co-workers.

After spherically symmetric situations, the most studied and best understood space-times are those that are stationary and have axial symmetry. In these situations the field equations can be reduced to a single equation involving a complex potential – the Ernst equation. It is now known that, with this, all possible stationary axisymmetric solutions can be generated in a finite number of steps using standard techniques. In their 1984 paper, Chandrasekhar and Ferrari showed that the main field equations for colliding plane waves can also be written as the same Ernst equation. In fact, it is then found that most of the techniques that have been developed for stationary axisymmetric space-times can also be applied to colliding plane waves. This has introduced considerable mathematical interest in the subject in recent years.

In fact colliding plane wave space-times have been found to have a surprisingly rich structure. Initially, it was widely believed that the collision of plane waves would necessarily produce a future space-like curvature singularity. This seemed to be implied by the focusing properties of plane waves. However, numerous counterexamples have subsequently been produced in which the curvature singularity is replaced by a Killing–Cauchy horizon. Extensions of the space-time through this horizon may, or may not, contain a space-like curvature singularity, or even a time-like curvature singularity which could be avoided by an observer travelling on a time-like world line. Recent research has clarified the singularity structure of most colliding plane wave space-times. These have been found to have a surprisingly rich variation.

In view of the recent advances, it is clearly time to present a comprehensive and unified review of the now vast literature on this topic. The purpose of this book is to provide such a review. Interesting lectures on this topic have been presented by Chandrasekhar (1986) and Ferrari (1989). However, in view of the considerable interest in the subject, a more thorough review is now required.

The first eight chapters of this book cover the background to the subject, presenting the field equations and a discussion of some qualitative aspects of their solution. A detailed discussion of the Khan–Penrose solution is included in this part, since it is the simplest solution and exhibits the general character of most colliding plane wave solutions. Further exact solutions for colliding plane gravitational waves are obtained and described in Chapters 9 to 14. The collision and interaction of electromagnetic waves is then considered in Chapters 15 to 19. The final chapters contain an attempt to summarize all related results for the collision of plane waves of different types and in non-flat backgrounds. A few general conclusions and some outstanding problems that still require attention are also indicated.

In the preparation of this book, I have been greatly assisted by a number of colleagues. I am extremely grateful to Chris Clarke, John Stewart and Sean Hayward for providing me with most helpful comments on the first draft of this work. This final version has been substantially expanded and seems to bear little resemblence to that initial draft. I am also very grateful to Parvinder Singh for reading through a late version, and to Roger Penrose and the American Physical Society for permission to copy Figure 4.1. Finally, I must record my debt to most of the authors of papers on colliding plane waves for regularly sending me preprints of their work prior to publication.

I have also benefited greatly from numerous discussions on colliding wave problems with many colleagues including Professor Chandrasekhar,

Alex Feinstein, Chris Clarke, Sean Hayward, Valeria Ferrari and Basilis Xanthopoulos. The views expressed in the book, however, are my own and I take full responsibility for any errors that it contains.

Loughborough J. B. G.
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