APPENDIX

COORDINATE SYSTEMS

Throughout the earlier chapters of this book, and frequently in the later chapters, it was found convenient to work in terms of two null coordinates u and v. It was found that one of the basic metric functions (6.23) could be written in the form

$$e^{-U} = f(u) + g(v) \tag{A.1}$$

where f and g are arbitrary functions.

As discussed in Section 7.2, however, it is found that the appropriate boundary conditions are only satisfied if these are monotonically decreasing functions whose leading terms take the form

$$f = \tfrac{1}{2} - (c_1 u)^{n_1} \ldots, \qquad g = \tfrac{1}{2} - (c_2 v)^{n_2} \ldots \tag{A.2}$$

where $n_1, n_2 \geq 2$. It is thus often appropriate to use f and g as decreasing null coordinates, at least in the interaction region IV.

Two more coordinate systems, both originally due to Chandrasekhar and Ferrari (1984) though in a different notation, were also introduced in equations (10.9–12). These involve a future pointing time-like coordinate t or ψ, and a space-like coordinate z or λ which are related by

$$t = \sin\psi, \qquad z = \sin\lambda. \tag{A.3}$$

These are related to f and g by

$$\begin{aligned} f &= \tfrac{1}{2}\cos(\psi+\lambda) = \tfrac{1}{2}\big(\sqrt{1-t^2}\sqrt{1-z^2} - tz\big) \\ g &= \tfrac{1}{2}\cos(\psi-\lambda) = \tfrac{1}{2}\big(\sqrt{1-t^2}\sqrt{1-z^2} + tz\big) \end{aligned} \tag{A.4}$$

or inversely by

$$\begin{aligned} t = \sin\psi &= \sqrt{\tfrac{1}{2}-f}\sqrt{\tfrac{1}{2}+g} + \sqrt{\tfrac{1}{2}-g}\sqrt{\tfrac{1}{2}+f} \\ z = \sin\lambda &= \sqrt{\tfrac{1}{2}-f}\sqrt{\tfrac{1}{2}+g} - \sqrt{\tfrac{1}{2}-g}\sqrt{\tfrac{1}{2}+f} \end{aligned} \tag{A.5}$$

so that

$$e^{-U} = f + g = \cos\psi\cos\lambda = \sqrt{1-t^2}\sqrt{1-z^2}. \tag{A.6}$$

The relations between various derivatives are frequently required. These are given by

$$\begin{aligned}
\frac{\partial}{\partial f} &= -\frac{1}{\sin(\psi+\lambda)}\left(\frac{\partial}{\partial\psi}+\frac{\partial}{\partial\lambda}\right)\\
\frac{\partial}{\partial g} &= -\frac{1}{\sin(\psi-\lambda)}\left(\frac{\partial}{\partial\psi}-\frac{\partial}{\partial\lambda}\right)\\
\frac{\partial}{\partial\psi} &= -\sqrt{\tfrac{1}{2}-f}\sqrt{\tfrac{1}{2}+f}\,\frac{\partial}{\partial f}-\sqrt{\tfrac{1}{2}-g}\sqrt{\tfrac{1}{2}+g}\,\frac{\partial}{\partial g}\\
\frac{\partial}{\partial\lambda} &= -\sqrt{\tfrac{1}{2}-f}\sqrt{\tfrac{1}{2}+f}\,\frac{\partial}{\partial f}+\sqrt{\tfrac{1}{2}-g}\sqrt{\tfrac{1}{2}+g}\,\frac{\partial}{\partial g}
\end{aligned} \qquad (A.7)$$

together with

$$\begin{aligned}
\frac{\partial}{\partial\psi} &= \sqrt{1-t^2}\,\frac{\partial}{\partial t}, & \frac{\partial}{\partial\lambda} &= \sqrt{1-z^2}\,\frac{\partial}{\partial z}\\
\frac{\partial}{\partial t} &= \frac{1}{\cos\psi}\,\frac{\partial}{\partial\psi}, & \frac{\partial}{\partial z} &= \frac{1}{\cos\lambda}\,\frac{\partial}{\partial\lambda}.
\end{aligned} \qquad (A.8)$$

It may also be noted that

$$\frac{\mathrm{d}t^2}{1-t^2}-\frac{\mathrm{d}z^2}{1-z^2}=\mathrm{d}\psi^2-\mathrm{d}\lambda^2=\frac{f'g'\,\mathrm{d}u\mathrm{d}v}{\sqrt{\tfrac{1}{2}-f}\sqrt{\tfrac{1}{2}-g}\sqrt{\tfrac{1}{2}+f}\sqrt{\tfrac{1}{2}+g}} \qquad (A.9)$$