Chemical Process Control

Topic 13

PID Controller Tuning

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Last time we saw: Bode criterion for stability

- What is system stability
- Bode stability criterion
  - Only applies to systems for which the gain and phase decrease continuously with frequency
  - System unstable if AR > 1 when $\varphi = 180°$
  - Gain margin (should be > 1.7)
  - Phase margin (must be > 30°)
- How to use direct substitution or Bode diagram for designing proportional gain K_c

Objectives for Today

- Tuning criteria
- Root-locus diagram
- Tuning controllers by pole positioning
- Tuning using the Ziegler-Nichols method for PID tuning
- Practical Issues

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Controller Design

Choose the controller **type**, and the **parameters** for the chosen controller, so that some objective criteria of closed-loop performance are satisfied

- Select controller type (guidelines in previous lectures)
- Involves selection of the proper values of K_c, τ_I, and τ_D.
- Affects control performance
- Affects controller reliability
- Controller tuning is, in many cases, a compromise between performance and reliability.

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Performance Criteria

There is no single best set of tuning parameters. Tuning depends on criteria!

- Stability criteria
- Steady state criteria (maximum admissible steady state offset)
- Dynamic Response Criteria
  - Minimum rise time
  - Minimum settling (response) time
  - Specify maximum overshoot
  - Specify maximum decay ratio (for oscillatory behavior)

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Performance Criteria

- Time integral performance criteria:
  - Integral Absolute Error (IAE): $IAE = \int_0^\infty |\varepsilon(t)|\, dt$
  - Integral Squared Error (ISE): $ISE = \int_0^\infty \varepsilon^2(t) dt$
- Other general criteria
  - Minimize variability
  - Remain stable for the worst disturbance upset (i.e., reliability) → robust stability
  - Avoid excessive variation in the manipulated variable

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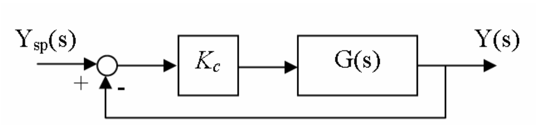
Example 1: Tuning via dynamic response criteria in few (one) point in time

- Determine the gain K_c of a P-only controller, so that the maximum overshoot for a unit step change in the setpoint is less than 40%.



$$G(s) = \frac{1}{(s+1)(1+0.5s)}$$

We will solve this in class

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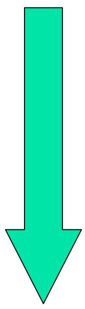
P-only Control

- For an open-loop overdamped process as K_c is increased the process dynamics goes through the following sequence of behavior
  - overdamped
  - critically damped
  - oscillatory
  - sustained oscillations
  - unstable oscillations



K_c increases

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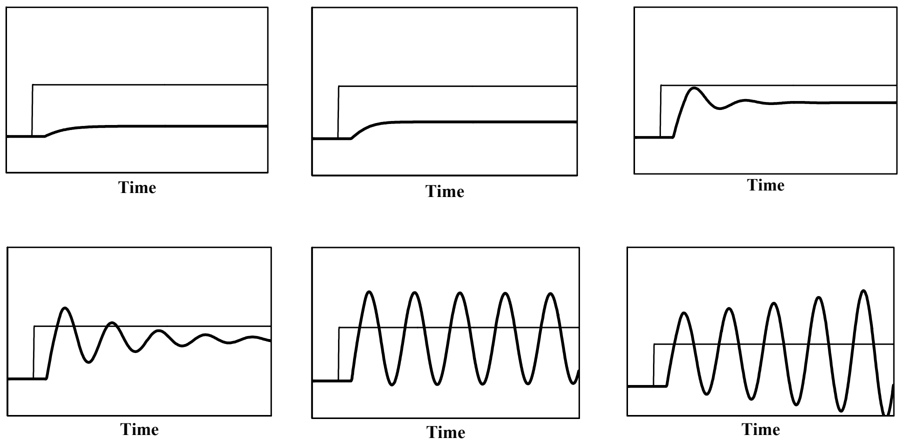
Dynamic Changes as K_c is increased for a FOPDT Process



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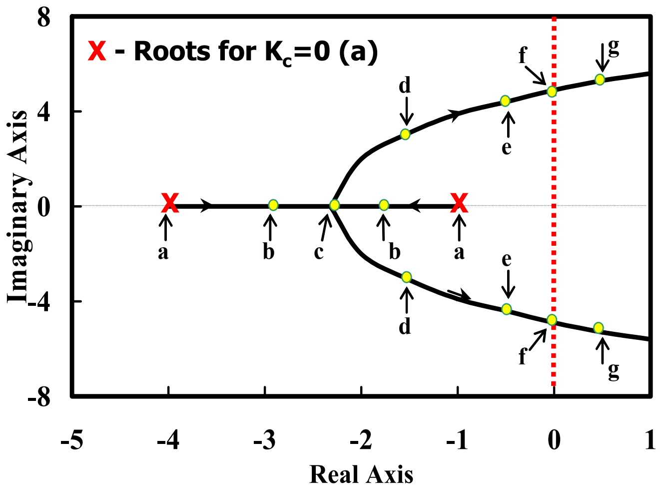
Root Locus Diagram (review) (K_c increases from a to g)



- [a, c] only real poles → no oscillations
- c – equal poles (critically damped)
- (c, f) – complex poles → stable with oscillations (oscillatory behaviour stronger from c to f
- f – poles with no real part → sustained oscillations
- > f – poles with positive real parts → oscillations with increasing amplitude (unstable)

- For physical systems Root Locus diagram is symmetrical with respect to real axis

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Controller Tuning by Pole Assignment (Pole Placement)

- Choose the closed-loop dynamic response by requiring the closed-loop poles of the system to be placed at specific locations and calculate the corresponding tuning parameters
- (recall: closed loop poles of a feedback system are the roots of the characteristic equation)
- Application of pole placement shows that the closed-loop damping factor and time constant are not independent
- Therefore, the decay ratio is a reasonable tuning criterion
- Only poles are chosen in this design → actual closed-loop response can be different based on existing zeros in the closed-loop transfer function

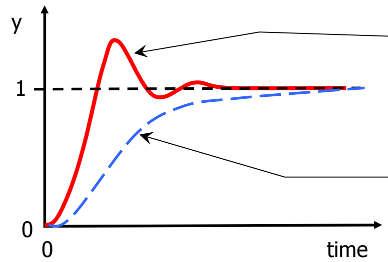
Controller Tuning by Pole Assignment (Pole Placement)

- A generalized controller (i.e., not only PID) can be derived by using pole placement.
- Generalized controllers are not generally used in industry because
  - Process models are not usually available
  - PID control is a standard function built into DCSs.

Example 2: Pole placement design

- Design a PID controller for a generic first order system so that the closed-loop system will have the response given by the reference trajectory described by:

$$G_{ref}(s) = \frac{1}{16s^2 + 4s + 1}$$



$\tau = 4$

$\varsigma = 0.5$

Underdamped trajectory with settling time determined by τ

We can choose other response types; e.g. critically damped

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Example 2: Pole placement design

- Characteristic equation: $G_p(s)\ G_a(s)\ G_c(s)\ G_s(s) + 1 = 0$
- Generic first order system: $G_p(s) = \dfrac{K}{\tau s + 1}$
- PID controller: $G_c(s) = K_c(1 + \dfrac{1}{\tau_I s} + \tau_D s)$
- Consider the only dominant dynamics is for the process:

$$G_p(s)G_c(s) \ + 1 = 0$$

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Example 2: Pole placement design

- Substituting in the equation gives the characteristic equation:

$$\tau_I \frac{KK_c\tau_D+\tau}{KK_c}s^2+\tau_I\frac{KK_c+1}{KK_c}s+1=0 \qquad G_{ref}(s)=\frac{1}{16s^2+4s+1}$$

Reference trajectory

$$\frac{\tau_I(KK_c\tau_D+\tau)}{KK_c}=16 \quad \text{and} \quad \frac{\tau_I(KK_c+1)}{KK_c}=4$$

- Solving:

$$K_c=\frac{\tau-4}{K(4-\tau_D)} \qquad \tau_I=\frac{4KK_c}{KK_c+1}$$

- By choosing K_c, τ_I and τ_D, we can position the roots (hence dynamic response) wherever we want in the complex plane

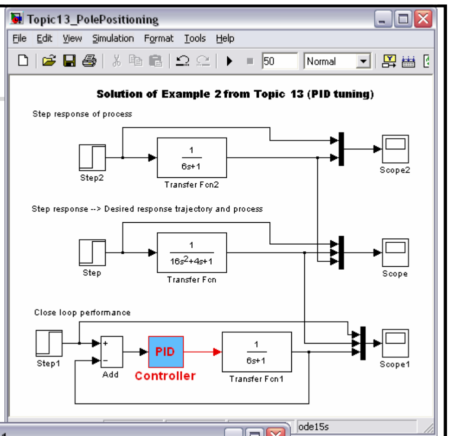
Example

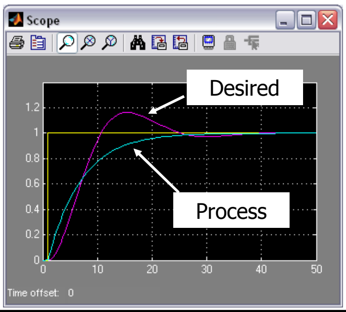
- Solving for the model:

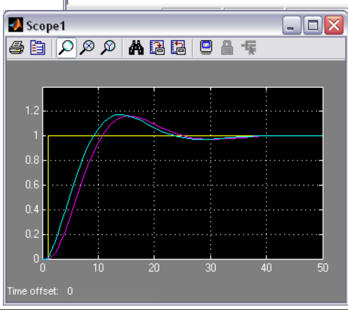
$$G_p(s)=\frac{1}{6s+1}$$

- Consider PI controller:

$$\tau_D=0;\ K_c=0.5;\ \tau_i=1.333$$







Classical Tuning Methods

- Example: Ziegler-Nichols and **many** others
- Are based on using the process model to determine the values of controller parameters that bring the closed-loop system to the verge of instability by
  - Simulation
  - Routh criterion
  - Direct substitution
- Use rules of thumb to reduce controller gain to provide a margin of safety away from closed-loop instability (e.g. see Bode stability margins from previous lecture)
- They are based on a preset tuning criterion (e.g., quarter decay ratio; i.e. the oscillation amplitudes dampen such that the decay ratio is < 0.25.

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Ziegler-Nichols Stability Margin Controller Tuning

❶ Using Proportional only control, determine $K_c = K_u$ (**ultimate controller gain**) which causes the system to be at the verge of instability

❷ Find the period of the sustained oscillations P_u (**ultimate period of oscillations)**:

$$P_u = \frac{2\pi}{\omega_{co}} \qquad \text{time/cycle}$$

Where ω_{CO} is the crossover frequency ($\varphi = 180°$)

Remember: ω_{CO} and K_u can be easily determined using the Bode stability criterion in conjunction with direct substitution or Bode diagram

❸ Find the Z-N settings for the controller from K_u and P_u using the rules summarized in the Table on the next slide

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Ziegler-Nichols Stability Margin Controller Tuning

- Published by Ziegler and Nichols in 1942
- Now are widely considered to be an industry standard

controller	K_c	τ_I	τ_D
P	$0.5\ K_u$	-	-
PI	$0.45\ K_u$	$P_u/1.2$	-
PID	$0.6\ K_u$	$P_u/2$	$P_u/8$

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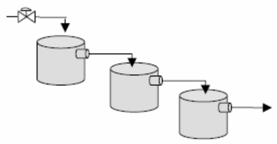
Example 3: Z-N tuning

- Design P, PI, and PID controllers for a three tank system for which the model is given by the following transfer function:

$$G_p(s) = \frac{6}{(2s+1)(4s+1)(6s+1)}$$



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Example 3: Z-N tuning

- Characteristic equation for the system with controller and process only:

$$G_p(s)G_c(s) \ +1=0$$

- Using the *method of substitution* we find the limiting value for a P-only controller (ultimate gain) controller for which the closed-loop system is at the verge of stability

Characteristic equation: $$\frac{6K_c}{(2s+1)(4s+1)(6s+1)}+1=0$$

or $$48s^3+44s^2+12s+(1+6K_c)=0$$

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Example 3: Z-N tuning

- By substituting $s = \omega j$ into the characteristic equation, we have:

$$-48j\omega^3-44\omega^2+12j\omega+(1+6K_c)=0$$

- Rearranging in the Cartesian form: **Re** + **Im***j*

$$(1+6K_c-44\omega^2)+(-48\omega^3+12\omega)j=0$$

- This equation can be satisfied if and only if both the real and imaginary parts are equal to zero:

$$-48\omega^3+12\omega=0$$

$$1+6K_c-44\omega^2=0$$

- Solving, gives:

$$\omega=0, \quad K_c=-1/6$$

$$\boxed{\omega=0.5, \quad K_c=1.67}$$

System is stable for $-1/6 < K_c < 1.67$, Hence, the ultimate gain is: $K_u = K_c = 1.67$

And the frequency of sustained oscillations at that point is: $\omega_{co} = 0.5\ rad/\min$

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Example 3: Z-N tuning

The ultimate period is given by:

$\omega_{co} = 0.5\ rad/\min$ → $P_u = \frac{2\pi}{\omega_{co}} = 12.56\ \min/cycle$

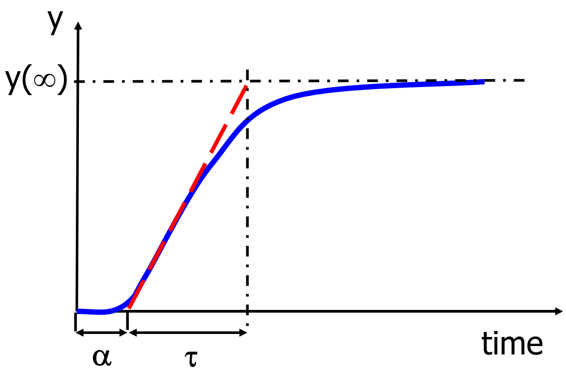
Ultimate gain is: $K_u = K_c = 1.67$

Z-N tuning will be:

- P: $K_c = 0.5K_u = 0.83$
- PI: $K_c = 0.45K_u = 0.75$; $\tau_I = P_u/1.2 = 10.5$ min
- PID: $K_c = 0.6K_u = 1.0$; $\tau_I = P_u/2 = 6.3$ min; $\tau_D = P_u/8 = 1.6$

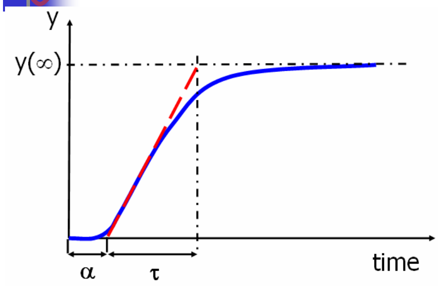
Controller tuning in time domain using approximate process model

- Approximate process model based on **<u>process reaction curve</u>** (obtained in open loop – no control)
- S-shaped reaction curve of any system can be approximated with a FOPDT transfer function



$$G_p(s) = \frac{Ke^{-\alpha s}}{\tau s + 1}$$

Controller tuning in time domain using approximate process model



$$G_p(s) = \frac{Ke^{-\alpha s}}{\tau s + 1}$$

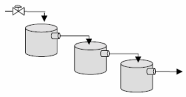
- Effective time delay (α): Draw tangent in inflection point, intersection with time axis gives α
- Effective Gain (K): $K = \frac{\text{ultimate value of the output}}{\text{magnitude of the step input}}$
- **Note that *y* is deviation variable**
- Effective time constant (τ): $\tau = \frac{y(\infty)}{slope}$

Ziegler-Nichols Approximate Model PID tuning rules

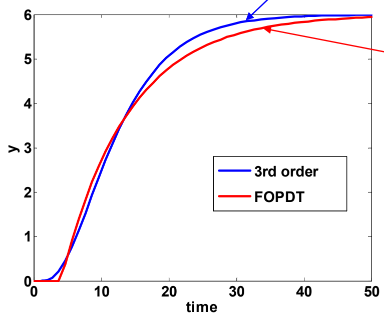
- Limited to models with $0.1<(\alpha/\tau)<1.0$
- There exist many other similar rules: e.g. Cohen-Coon, etc.

controller	K_c	τ_I	τ_D
P	$\frac{1}{K}\left(\frac{\tau}{\alpha}\right)$	–	–
PI	$\frac{0.9}{K}\left(\frac{\tau}{\alpha}\right)$	3.33α	–
PID	$\frac{1.2}{K}\left(\frac{\tau}{\alpha}\right)$	2.0α	0.5α

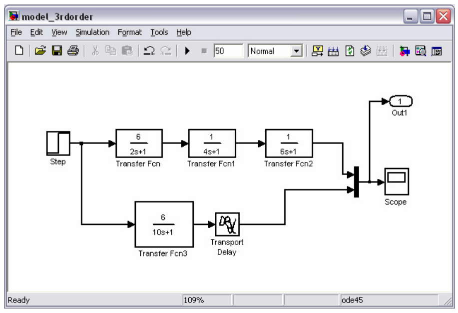
Example 4



$$G_p(s) = \frac{6}{(2s+1)(4s+1)(6s+1)}$$



$$G_{\text{model}}(s) = \frac{6e^{-3s}}{10s+1}$$

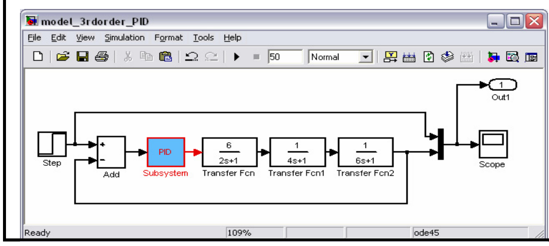


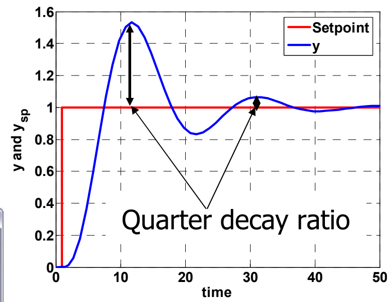
Example 4

$$G_{\text{model}}(s) = \frac{6e^{-3s}}{10s+1}$$

- Z-N approximate model PID:
  - K = 6
  - α = 3
  - τ = 10

$$K = \frac{1.2}{6}\left(\frac{10}{3}\right) = 0.6667$$
$$\tau_I = 2 \times 3 = 6$$
$$\tau_D = 0.5 \times 3 = 1.5$$





Summary

- Controller tuning depends on criteria
- We can design the controller to have a particular closed-loop response (pole positioning)
- Classical tuning criteria based on stability margin: Ziegler-Nichols method
- Tuning based on approximate model (FOPDT) obtained using process reaction curve

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